

AP Calculus AB
Test: Applications of the Integral
Instructor's Version

Free-Response Question 1 — no calculator (15 points)

1. The high temperature at Mt. Charles was recorded every two days in the table below.

t (days)	0	2	4	6	8	10
High temperature $^{\circ}\text{F}$	50	52	54	50	50	48

- A. Approximate the average high temperature over the time interval $0 \leq t \leq 10$ days using a trapezoidal approximation with subintervals of length $\Delta t = 2$ days. (5 points)

Answer:

$$\frac{1}{10} \cdot \frac{2}{2} [50 + 2(52 + 54 + 50 + 50) + 48] = 51^{\circ}\text{F}$$

- 1 point: divided by 10
- 2 points: trapezoidal rule
- 1 point: answer
- 1 point: units

- B. If the function $f(t) = -.1x^2 + x + 50$ is used to model the high temperature at day t , find the average value of f over the time interval $0 \leq t \leq 10$ days. (5 points)

Answer:

$$\frac{1}{10} \int_0^{10} f(t) dt = \frac{1}{10} \left[\frac{-1}{30} x^2 + \frac{1}{2} x^2 + 50x \right]_0^{10} = 51\frac{2}{3} \text{ or } 51.667^{\circ}\text{F}$$

- 1 point: divided by 10
- 1 point: integrand and limits
- 2 points: antiderivative
- 1 point: answer

- C. Using the function f from part (B), find $f'(6)$. Explain the meaning of your answer. (5 points)

Answer:

$$f'(x) = -.2x + 1$$
$$f'(6) = -.2$$

The temperature is decreasing at a rate of $.2^{\circ}\text{F}$ per day when $t=6$ days.

- 1 point: derivative $f'(x) = -.2x + 1$
- 1 point: $f'(6) = -.2$
- 2 points: interprets negative sign as decreasing temperature
- 1 point: units ($^{\circ}\text{F}$ per day)

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Free-Response Question 2 — with calculator (15 points)

2. The region R is bounded by the x -axis, $x = 0$, $x = 4$, and $y = \frac{x^2}{4}$.

A. Find the area of R . Give the exact answer. (3 points)

Answer:

$$A = \int_0^4 \frac{x^2}{4} dx = \frac{x^3}{12} \Big|_0^4 = \frac{16}{3}$$

1 point: limits
1 point: integrand
1 point: answer

B. Find the value of k such that the line $x = k$ divides the region R into two regions of equal area. (4 points)

Answer:

$$\int_0^k \frac{x^2}{4} dx = \frac{8}{3}$$

$$\frac{x^3}{12} \Big|_0^k = \frac{8}{3}$$

$$\frac{k^3}{12} = \frac{8}{3} \Leftrightarrow k = \sqrt[3]{32}$$

$$k = 2\sqrt[3]{4} \approx 3.175$$

1 point: sets integral = 1/2 answer from part (A)
2 points: antiderivative
1 point: answer

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- C. Find the volume of the solid generated when R is revolved about the x -axis. Give the exact answer. (4 points)

Answer:

$$V = \pi \int_0^4 \left(\frac{x^2}{4} \right)^2 = \pi \left(\frac{x^5}{180} \right)_0^4 = \frac{256\pi}{45}$$

1 point: multiplied by π
2 points: integrand with limits
1 point: answer

- D. The vertical line $x=m$ divides the region R into two regions such that when the two regions are revolved about the x -axis, they generate solids with equal volumes. Find the value of m . Round to three decimal places. (4 points)

Answer:

$$\pi \int_0^m \frac{x^4}{16} dx = \frac{128\pi}{45}$$

$$\text{Let } Q = \pi \left(\frac{x^5}{180} \right)_0^m = \frac{128\pi}{45}$$

$$\frac{m^5}{180} = \frac{128}{45} \Leftrightarrow m^5 = 512$$

$$m = \sqrt[5]{512} = 2\sqrt[5]{16} \approx 3.482$$

1 point: integrand with limits
1 point: multiplied by π
1 point: equated to 1/2 answer from part (C)
1 point: answer